SECTION I, Part A

Time - 60 **Minutes**

Number of questions - 30

A CALCULATOR MAY NOT BE USED IN THIS PART OF THE EXAMINATION.

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1. $\int_1^e \left(\frac{x^2 - x + 1}{x^2}\right)\ dx$

(A) $e - \frac{1}{e}$

(B) $e^2 - e$

(C) $e - 1 - \frac{1}{e}$

(D) $e - 3 - \frac{1}{e}$

2. If $f(x) = \cos(e^{-\tan x})$, then $f'(x) =$

(A) $-\sin(e^{-\tan x})$

(B) $e^{-\tan x} \sin x \sec^2(e^{-\tan x})$

(C) $e^{-\tan x} \sin(e^{-\tan x})$

(D) $e^{-\tan x} \sin(e^{-\tan x}) \sec^2 x$

3. If $F(x)=\int_0^x t^3+2\ dt$, then $F(2)=$

(A) 4

(B) 8

(C) 10

(D) 12

4. If $f(x)=2\cos^2 x+\tan x$, then $f'\left(\frac{\pi}{6}\right)=$

(A) $4-\sqrt{3}$

(B) $\frac{4-3\sqrt{3}}{3}$

(C) $\frac{4+3\sqrt{3}}{3}$

(D) $4+\sqrt{3}$

5. At which of the following points is the graph of $f(x) = x^5 - 10x^3$ increasing and concave up?

(A) $(-\infty, -\sqrt{6})$

(B) $(-\sqrt{3},0)$

(C) $(0,\sqrt{6})$

(D) $(\sqrt{6}, \infty)$

6. Which of the following are antiderivatives of $f(x) = \sin^2 x \cos x + 2 \cos x \sin x$?

I. $F(x) = \dfrac{\sin^3 x}{3} + \sin^2 x$

II. $F(x) = \dfrac{2 + \sin^3 x}{3} - \cos^2 x$

III. $F(x) = \dfrac{\cos^3 x}{3} - \dfrac{\cos^2 x}{2}$

(A) I only

(B) II only

(C) II and III

(D) I and II

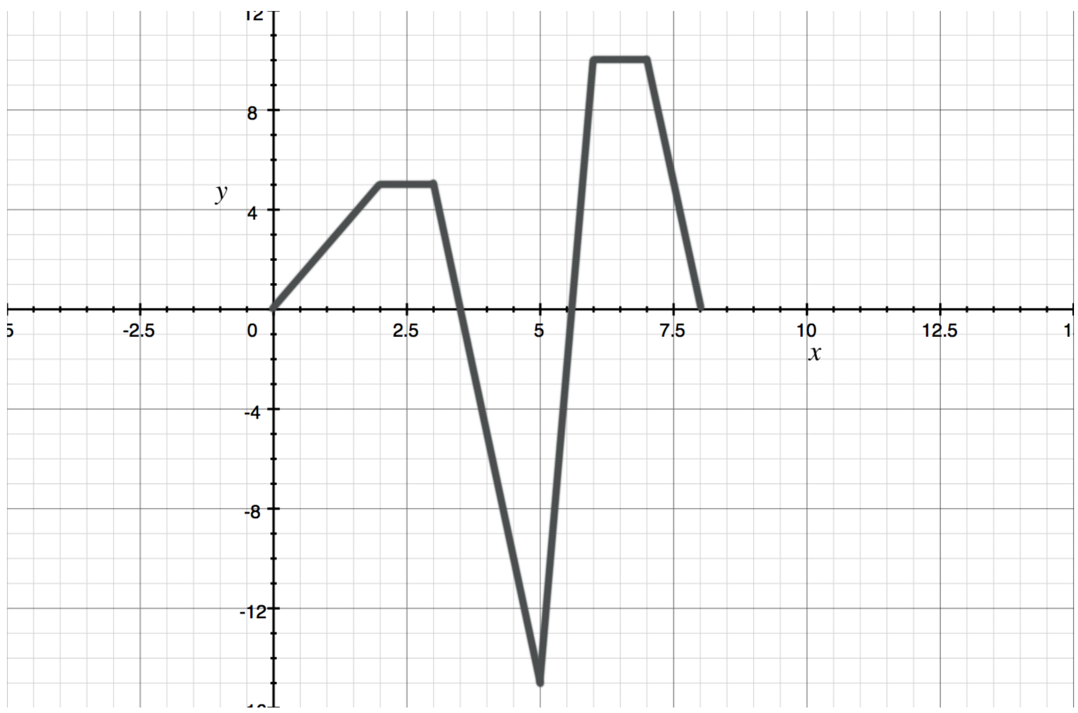
7. $\frac{d}{dx}(\cos(e^{3x^2})) =$

(A) $-6x\sin(e^{3x^2})$

(B) $-6xe^{3x^2}\sin x$

(C) $-e^{6x}\sin(e^{3x^2})$

(D) $-6xe^{3x^2}\sin(e^{3x^2})$



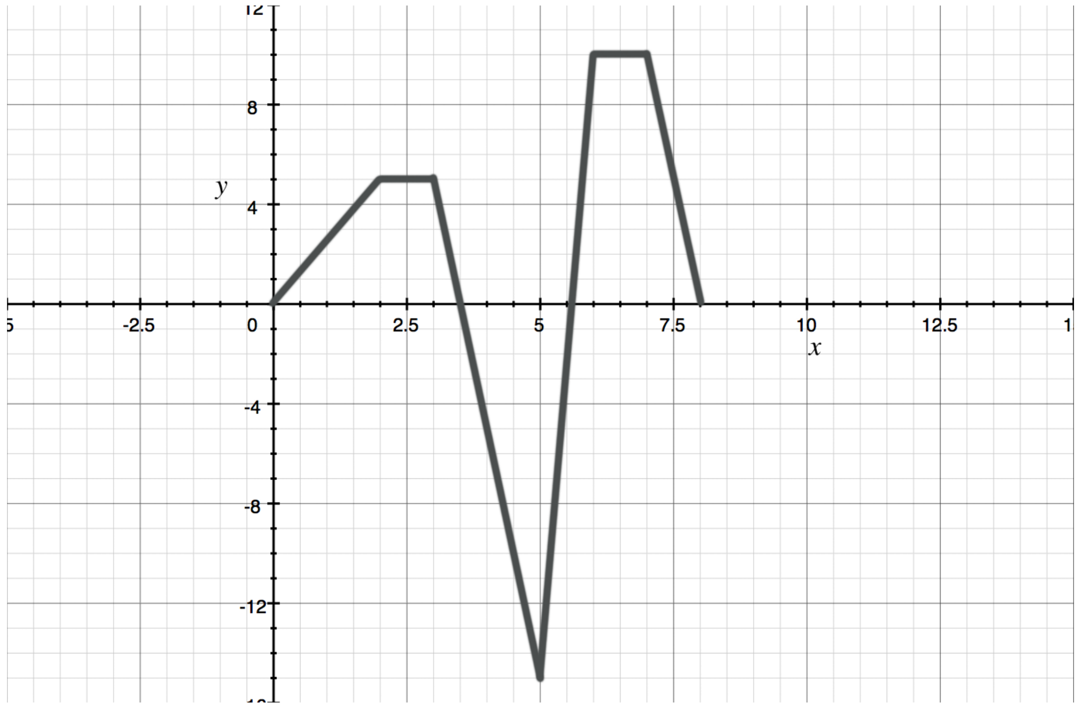
8. The graph above shows the velocity of a moving object as a function of time. At what time has the object reached its maximum speed?

(A) $t = 0$

(B) $t = 2$

(C) $t = 5$

(D) $t = 6$



9. The graph above shows the velocity of a moving object as a function of time. Over which interval does the object have a zero acceleration?

(A) $[0,2]$ only

(B) $[3,5]$ only

(C) $[2,3]$ and $[6,7]$

(D) $[5,6]$ only

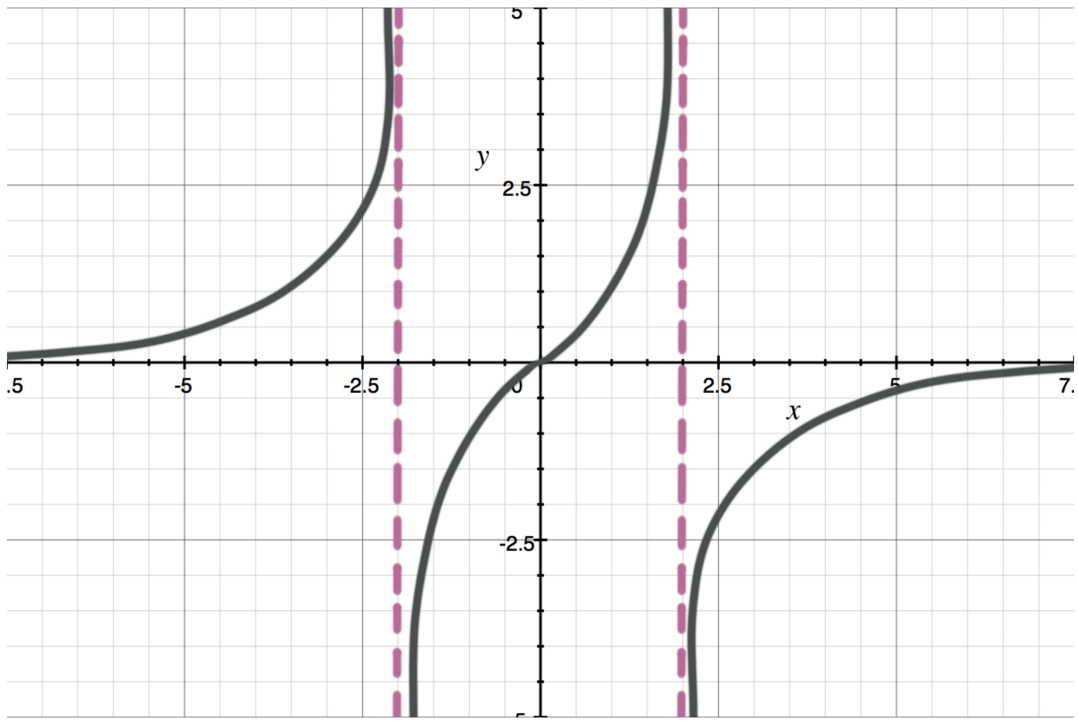
10. An equation of the line tangent to $y = \cos 2x + 3\tan 2x$ at $\left(\frac{\pi}{2}, 3\right)$ is

(A) $6x - y = 3\pi - 3$

(B) $4x - y = 2\pi - 3$

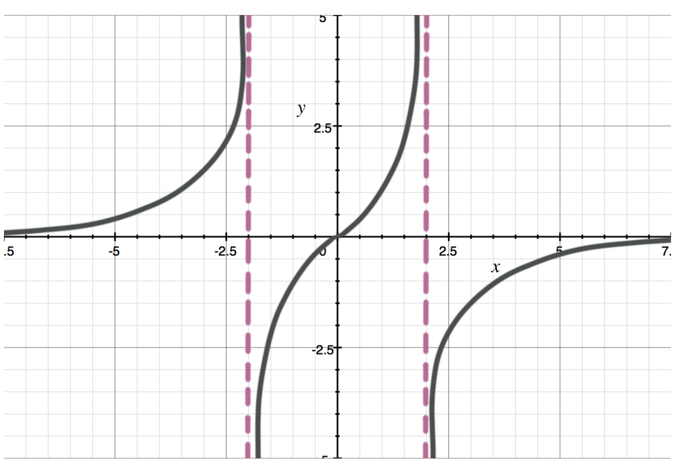
(C) $6x + y = 3\pi + 3$

(D) $2x + y = \pi + 3$



11. The graph of the derivative of f is given above. Which of these graphs could be the function f?

(A)



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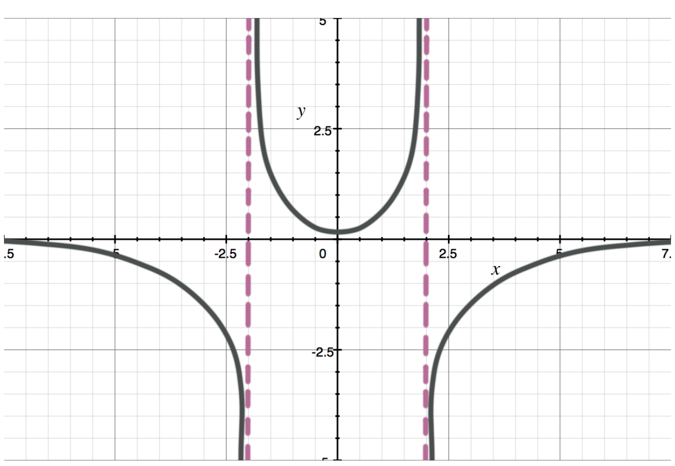
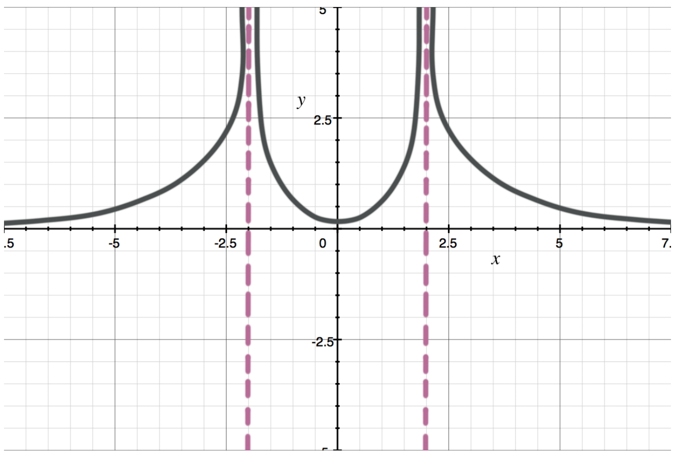
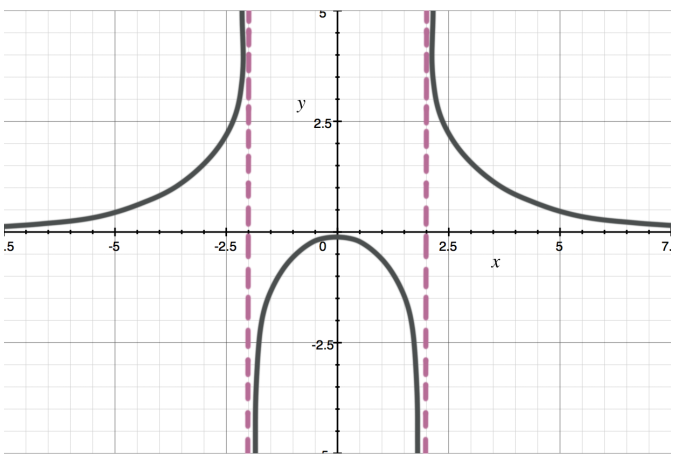


(C) 

(D) 

12. The function f is given by $f(x) = x^4 - 10x^3 + 25x^2 - 36$. All of these statements are true, except

(A) -1 and 2 are zeros of f.

(B) $f'(0) = 0$

(C) $(0, -36)$ is a local maximum of f.

(D) $(5, -36)$ is a local minimum of f.

13. Let f be a function defined for all real numbers x. If $f'(x) = \frac{|16 - x^2|}{x - 4}$, then f is decreasing on which of the following intervals?

(A) $(-\infty, 4)$

(B) $(-4, 4)$

(C) $(-4, \infty)$

(D) $(4, \infty)$

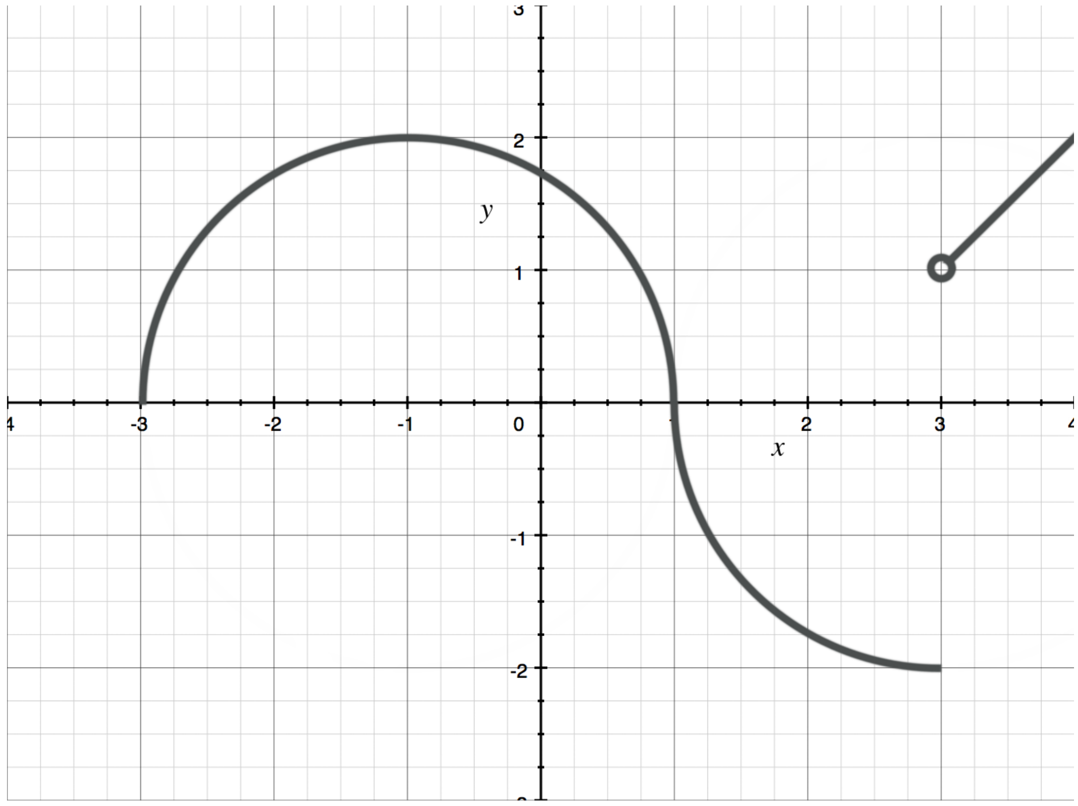
14. Let f and g be twice differentiable functions such that $g'(x) \leq 0$ for all x in the domain of f. If $h(x) = g(f'(x))$ and $h'(2) = -3$, then which of the following is true at $x = 2$?

(A) h is concave down.

(B) f is concave up.

(C) g is concave up.

(D) f is increasing.



15. The graph of the function f shown in the figure above has a vertical tangent at the point $(1,0)$ and horizontal tangents at the points $(-1,2)$ and $(3,-2)$. For what values of x, $-3 < x < 4$, is f not differentiable?

(A) 0 only

(B) 1 and 3 only

(C) -1 and 3 only

(D) -1, 1, and 3

16. What is the area of the region bounded by the curves $y = x^2 - x - 12$ and $y = -x^3$ from $x = 0$ to $x = 2$?

(A) $\frac{46}{3}$

(B) $\frac{58}{3}$

(C) $\frac{70}{3}$

(D) $\frac{98}{3}$

17. Determine $\frac{dy}{dx}$ for the curve defined by $3x^2 + y^3 = 3xy$.

(A) $\frac{y - x^2}{y^2 - x}$

(B) $\frac{2x - y}{y^2 - x}$

(C) $\frac{2x}{y - y^2}$

(D) $\frac{y - 2x}{y^2 - x}$

18. Evaluate the integral $\int x^3 + \frac{x}{4} + \frac{2}{x} + 5\ dx$.

(A) $\frac{x^4}{4} + \frac{x^2}{8} + 2\ln x + 5x$

(B) $\frac{x^4}{4} + \frac{x^2}{8} + 2\ln x + 5x + C$

(C) $\frac{x^4}{4} + \frac{x^2}{2} - \frac{4}{x^2} + 5x + C$

(D) $x^3 + \frac{x}{4} + \frac{2}{x} + 5$

19. Find all inflection points for the polynomial $f(x) = x^4 - 6x^3 + 12x^2$.

(A) $(1,7)$ and $(2,-16)$

(B) $(2,-16)$

(C) $(1,7)$ and $(2,16)$

(D) There are no inflection points.

20. What is the average value of $y = \frac{3}{x}$ over $[1,e]$?

(A) 1

(B) 3

(C) $\frac{3}{e-1}$

(D) $\frac{1}{e-1}$

21. $\lim_{x\to\infty} \frac{x^2 - 4x + 5}{x^2 - x^3 + 8} =$

(A) -1

(B) 0

(C) 1

(D) Does not exist.

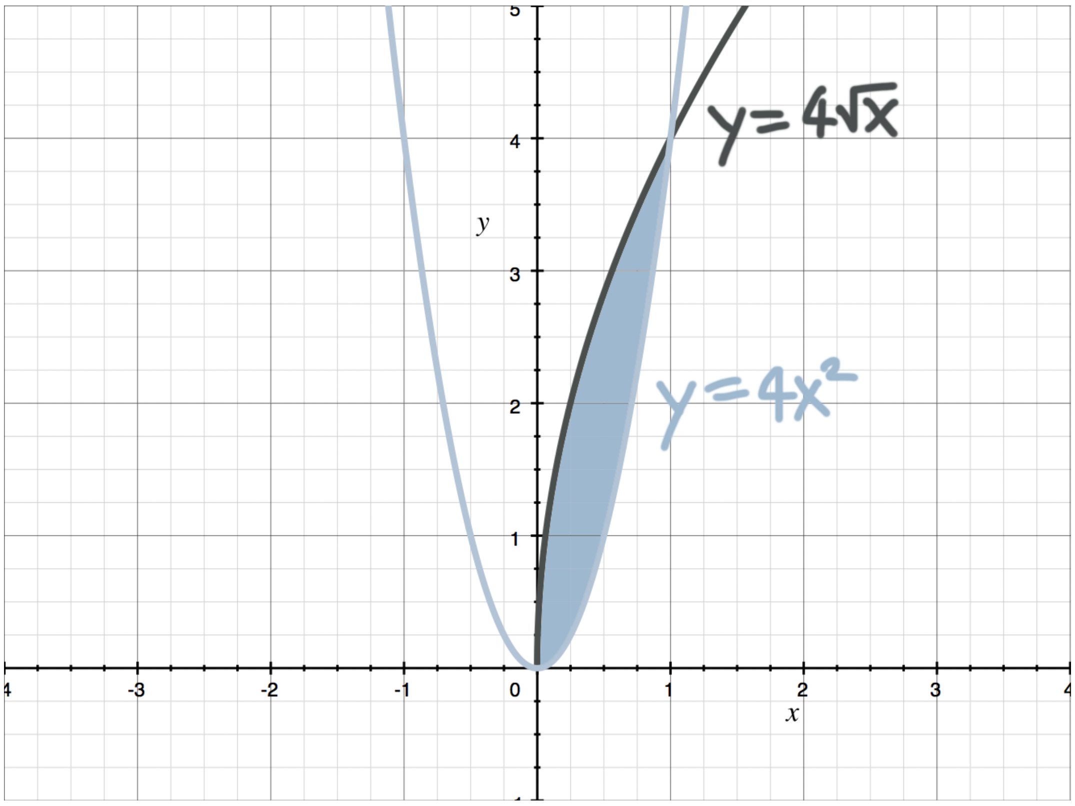
22. The graph of $f(x) = \frac{2 + x}{x^2 - 4}$ is concave up over which of the following interval(s)?

(A) $(-\infty, 2)$

(B) $(-\infty, -2)$ and $(-2, 2)$

(C) $(2, \infty)$

(D) $(-\infty, \infty)$



23. The area of the shaded region in the diagram above is equivalent to

(A) $\int_0^4 4x^2 - 4\sqrt{x}\ dx$

(B) $\pi \int_0^4 16x^4 - 16x\ dx$

(C) $\int_0^1 4\sqrt{x} - 4x^2\ dx$

(D) $2\pi \int_0^1 \sqrt{x} - x^2\ dx$

24. $\lim_{h \to 0} \frac{3\cos 2\left(\frac{\pi}{4}+h\right) - 3\cos\frac{\pi}{2}}{h} =$

(A) -6

(B) -3

(C) 0

(D) 2

25. $\int_0^{\frac{\pi}{2}} \sin^3 x \cos x \ dx =$

(A) $-\frac{1}{4}$

(B) $\frac{1}{4}$

(C) $\frac{\pi^4}{64}$

(D) $\frac{\pi^4}{16}$

26. A particle's position is given by $s(t) = 2\sin t + \frac{2t}{\pi} + 4$. The average velocity of the particle over $\left[0, \frac{3\pi}{2}\right]$ is

(A) $1 + \frac{8}{3\pi}$

(B) $\frac{10}{3\pi}$

(C) $\frac{2}{3\pi}$

(D) 3π

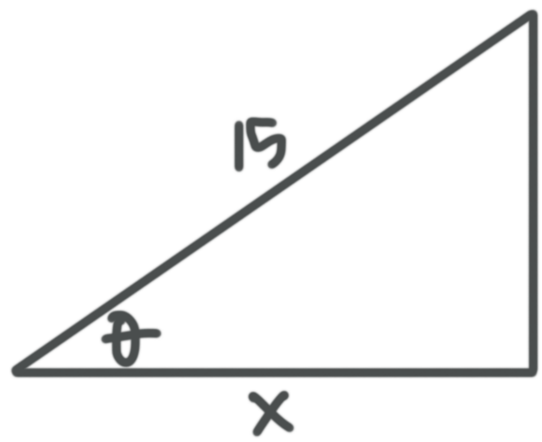
27. If $f(x) = \begin{cases} 4\ln x^2 & 0 < x < 2 \\ x^2 \ln 4 & x \geq 2 \end{cases}$, then $\lim_{x\to 2} f(x) =$

(A) 4

(B) $\ln 4$

(C) $4\ln 4$

(D) The limit does not exist.



28. In the triangle shown above, θ is at decreasing at a constant rate of 5 radians per minute. What is the rate of change of x, in units per minute, when $x=9$ units?

(A) -60

(B) 36

(C) 45

(D) 60

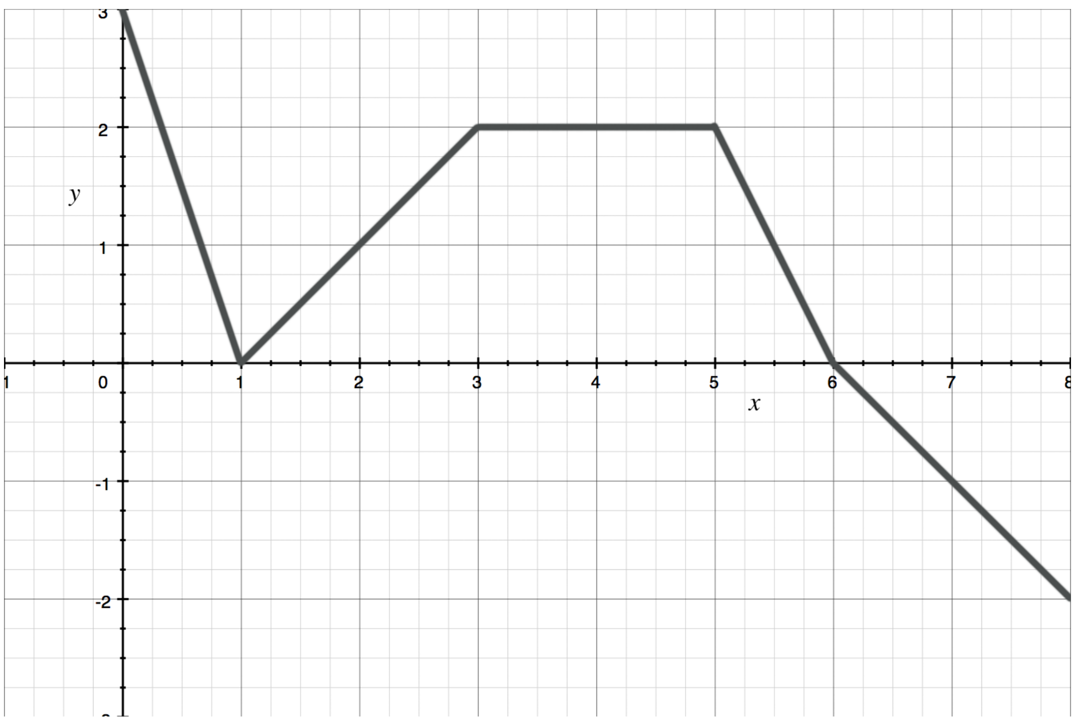
29. If $f(x)=\dfrac{\cos x^3}{e^{2x^2}}$, then $f'(x)=$

(A) $\dfrac{-3x\sin x^3}{4e^{2x^2}}$

(B) $\dfrac{3x^2\sin x-4x\cos x^3}{e^{2x^2}}$

(C) $\dfrac{3x^2\sin x^3+4x\cos x^3}{e^{4x^2}}$

(D) $\dfrac{-3x^2\sin x^3-4x\cos x^3}{e^{2x^2}}$



30. The graph of the derivative of f is shown above. If $f(0) = 0$, then which of the following statements is true?

(A) $f(8) < f(6) < f(1) < f(3)$

(B) $f(8) < f(1) < f(6) < f(3)$

(C) $f(1) < f(3) < f(6) < f(8)$

(D) $f(1) < f(3) < f(8) < f(6)$

END OF PART A, SECTION I

SECTION I, Part B

Time - 45 **Minutes**

Number of questions - 15

A GRAPHING CALCULATOR IS REQUIRED FOR SOME QUESTIONS ON THIS PART OF THE EXAMINATION.

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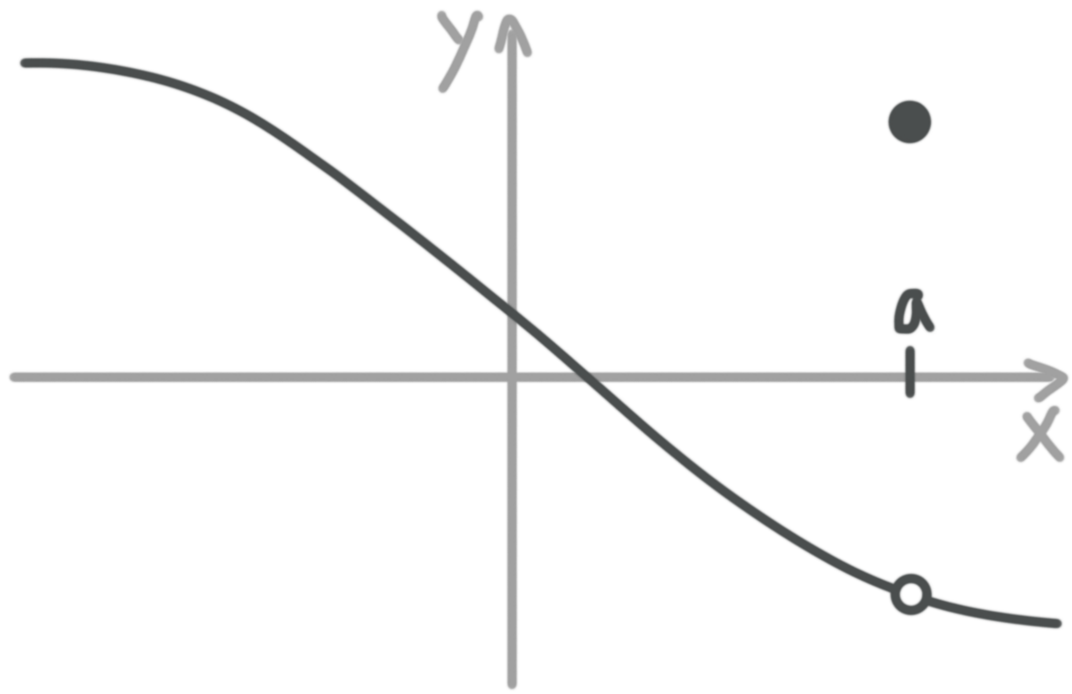
31. Which of the following is an equation for a line tangent to the graph of $f(x) = \ln(3x + 2)$ when $f'(x) = 5$?

(A) $y = x + 1.823$

(B) $y = 5x + 1.823$

(C) $y = 5x$

(D) $y = x - 2.847$



32. The graph of a function f is shown above. Which of the following statements about f is false?

(A) $x = a$ is in the domain of f

(B) $\lim_{x\to a^+} f(x)$ is equal to $\lim_{x\to a^-} f(x)$

(C) f is continuous at $x = a$

(D) $\lim_{x\to a} f(x)$ exists

33. If g is a differentiable function such that $g(x) > 0$, for all real numbers x, and if $f'(x) = (x^2 + 2x - 8) \cdot g(x)$, which of the following is true?

(A) f has a relative maximum at $x = -4$ and a relative minimum at $x = 2$.

(B) f has a relative maximum at $x = 2$ and a relative minimum at $x = -4$.

(C) f has a relative minimum at $x = 2$ and $x = -4$.

(D) f has a relative maximum at $x = 2$ and $x = -4$.

34. The radius of a circle is increasing at a constant rate of 0.15 centimeters per second. What is the rate of change of the area of the circle, in square centimeters per second?

(A) $0.3\pi C$

(B) $0.15C$

(C) $-0.15C$

(D) $-\frac{0.3C}{\pi}$

35. Let $f(x) = \frac{|x^2 - 4|}{x + 2}$. Which of these statements is(are) false?

I. f is not continuous at $x = 2$.

II. f is differentiable at $x = -2$.

III. f has a local minimum at $x = 2$.

(A) I only

(B) I and II only

(C) II and III only

(D) I, II, and III

36. Let $F(x)$ be an antiderivative of $\frac{(\ln 2x)^4}{x}$. If $F\left(\frac{1}{2}\right) = 2$, then $F(5) =$

(A) 2.160

(B) 12.945

(C) 14.945

(D) 8.473

37. What is the area of the region bounded by $y = \sin\frac{x}{2}$, $y = -x + 2$, and the y-axis?

(A) 1.255

(B) 1.350

(C) 1.368

(D) 3.222

38. When the region enclosed by the graphs of $y = 2x$ and $y = 6x - x^2$ is revolved about the y-axis, what is the volume of the resulting solid?

(A) 10.667

(B) 67.021

(C) 134.041

(D) 544.545

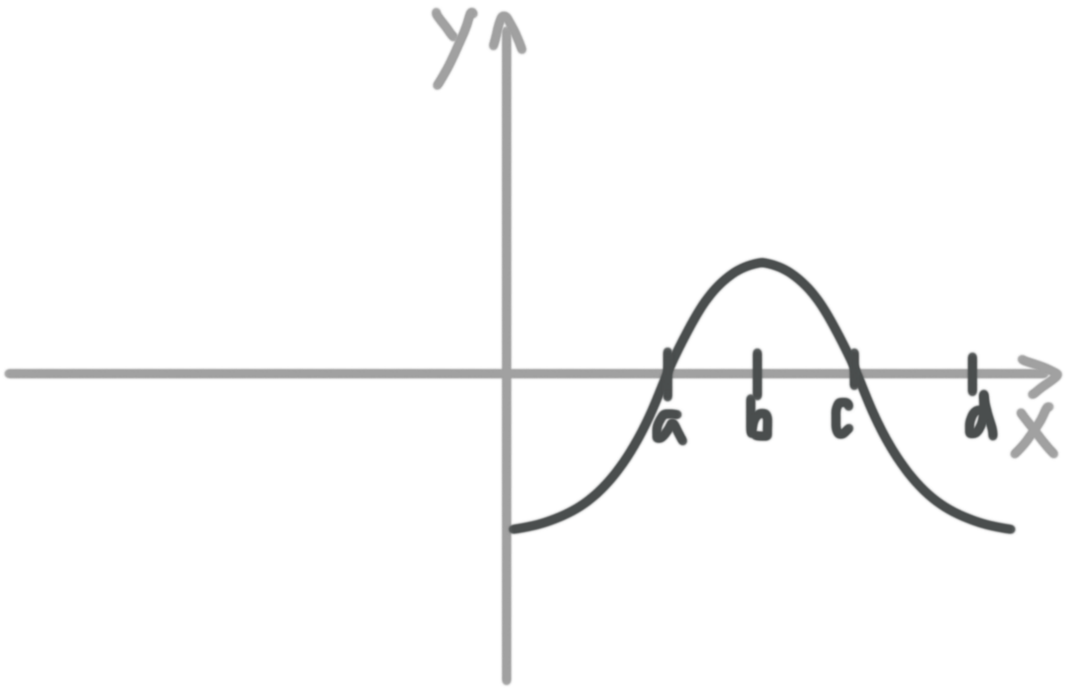
39. Let y be the function given by $f(x) = 2e^{3x}$ and let g be the function given by $g(x) = 5x^3 - 2x$. At what value of x do the graphs of f and g have parallel tangent lines?

(A) -0.366

(B) -0.478

(C) -0.251

(D) -0.414



40. The graph of f is shown above. If $g(x) = \int_a^x f(t)\, dt$, for what value of x does $g(x)$ have a relative minimum?

(A) a

(B) b

(C) c

(D) d

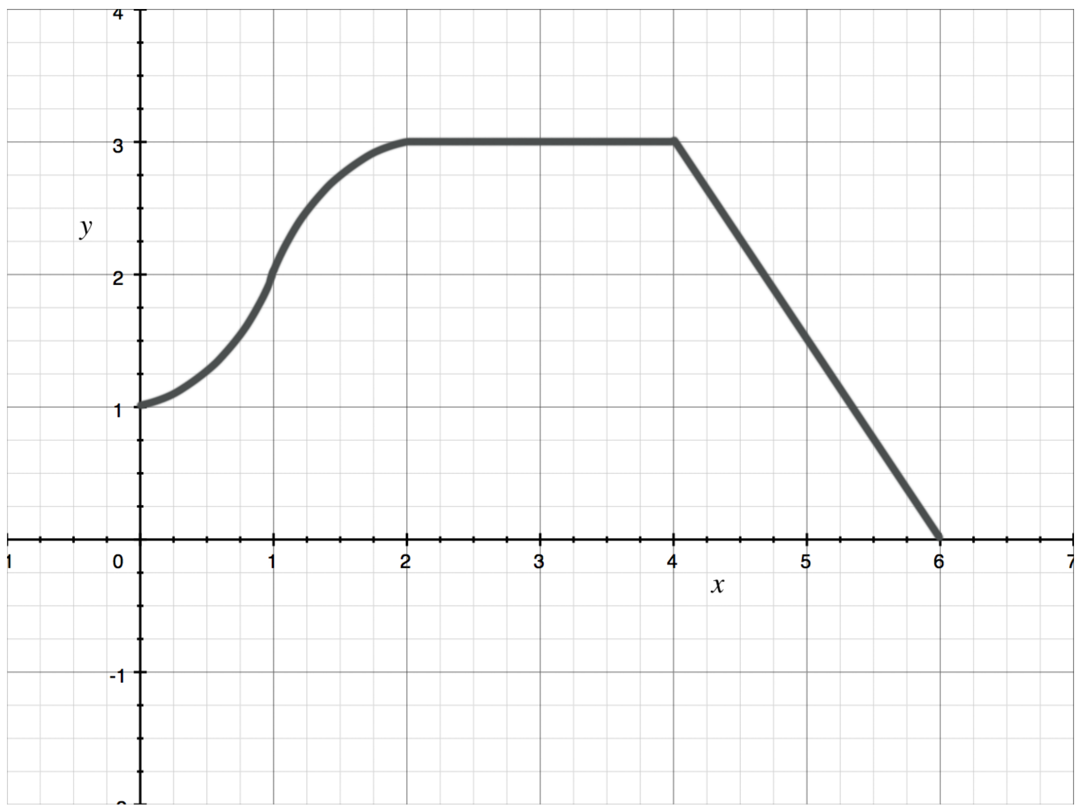
41. The graph of the function $y = 2x^3 + 3x^2 + 5 - 2\sin x$ changes concavity at $x =$

(A) -0.593

(B) -0.430

(C) 0.257

(D) 0.484



42. The graph of f is shown in the figure above. If $\int_0^2 f(x)\ dx = 4.2$ and $F'(x) = f(x)$, then $F(6) - F(0) =$

(A) 1.3

(B) 10.2

(C) 13.2

(D) 16.2

x	0	2	4	6	8	10	12
f(x)	5.0	2.3	3.1	1.0	4.5	6.2	4.6

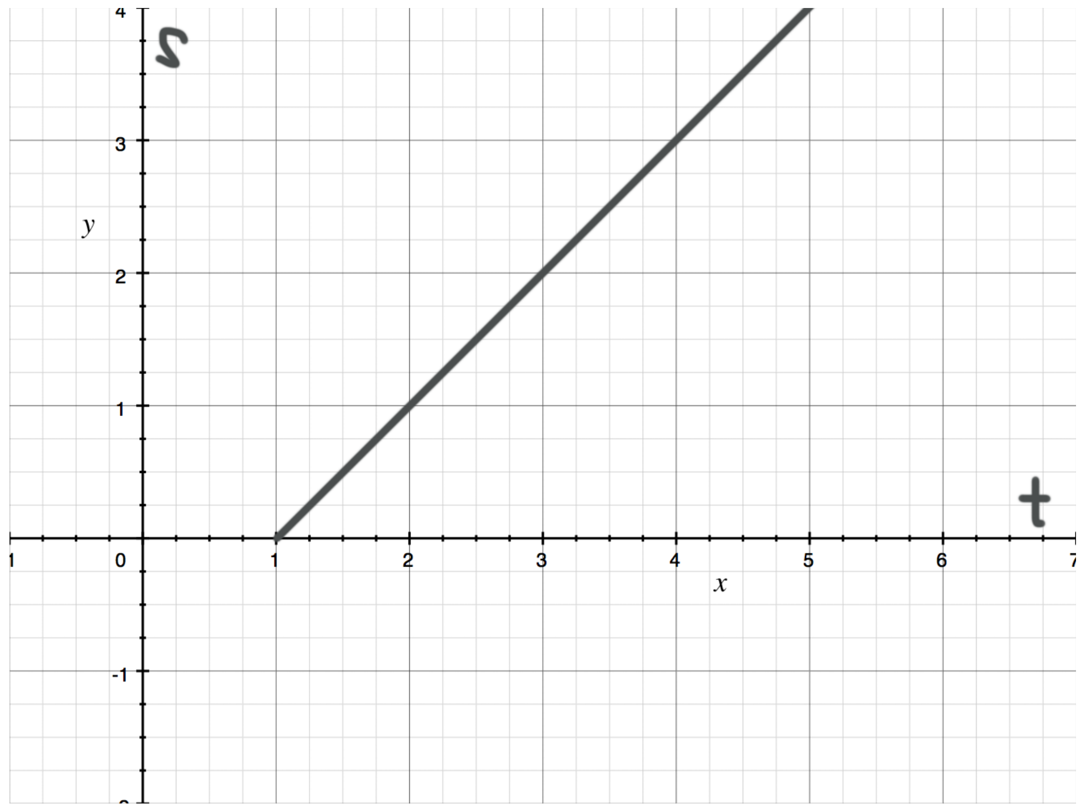
43. A table of values for a continuous function f is shown above. If six equal subintervals of $[0,12]$ are used, which of the following is equivalent to a right-hand Riemann sum approximation for $\int_0^{12} f(x)\, dx$?

(A) 21.7

(B) 43.4

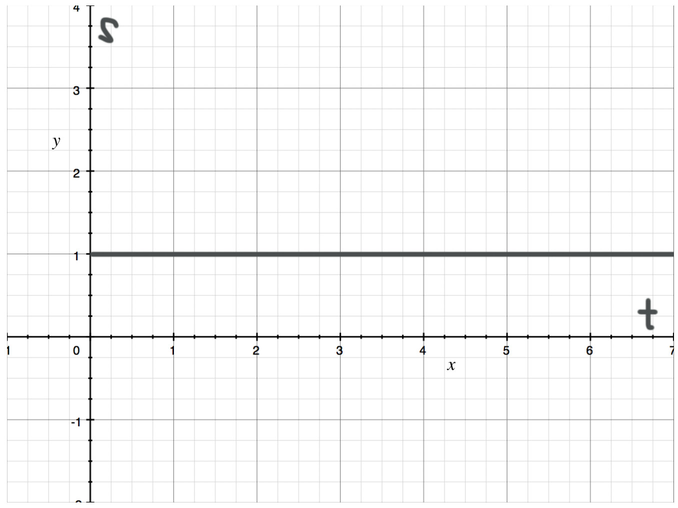
(C) 44.2

(D) 53.4

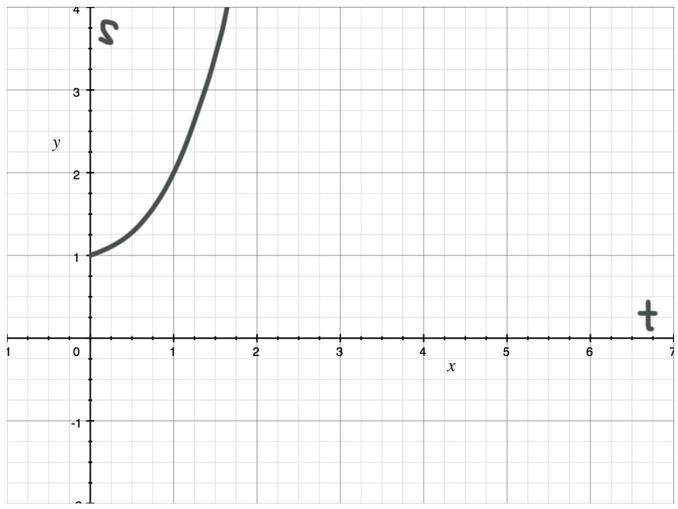


44. The graph of the distance $s(t)$ of the particle as a function of time t is shown above. Which of the following could be the graph of the velocity $v(t)$ of the particle?

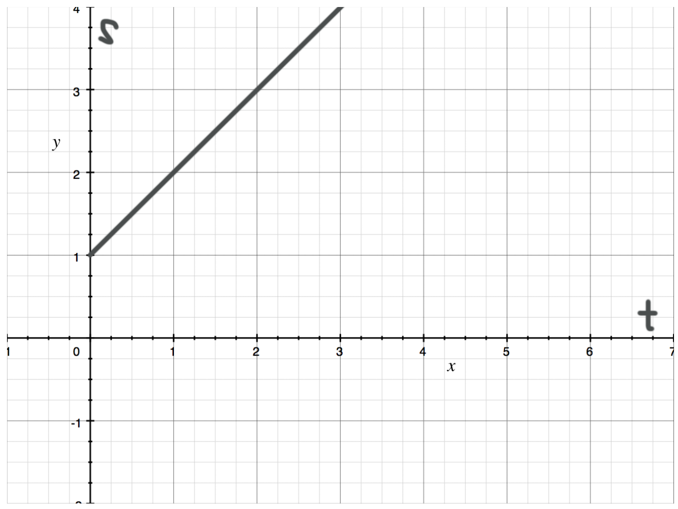
(A)



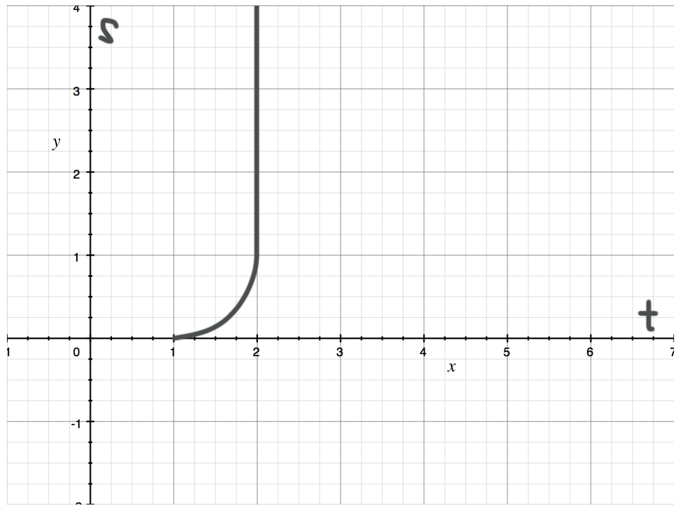
(B)



(C)



(D)



45. If $a \neq 0$, then $\lim_{x \to a} \frac{x^3 - a^3}{x^2 - a^2} =$

(A) $\frac{1}{2}a$

(B) $\frac{3}{2}$

(C) 0

(D) $\frac{3}{2}a$

END OF PART B, SECTION I

SECTION II, PART A

Time - 30 **Minutes**

Number of problems - 2

A GRAPHING CALCULATOR IS REQUIRED FOR SOME PROBLEMS OR PARTS OF PROBLEMS.

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1. A water tank has the shape of an open circular cone. The height of the water tank is 20 cm and the radius of the opening is 10 cm. At height h, the radius of the water tank is given by $r = \frac{1}{10}(h^2 - 6)$, where $0 \le h \le 20$.

a. Find the average value of the radius of the water tank.

b. Find the volume of the water tank.

c. Find the rate of change of the volume of water in the water tank with respect to time and when $h = 5$ cm, if water is evaporating so that its depth h is changing at the constant rate of $-\frac{2}{5}$ cm/hr.

2. A particle moves along the x-axis so that its velocity v at time t, for $0 \le t \le 7$, is given by $v(t) = \ln(t^2 - t + 1)$. The particle is at position $x = 5$ at time $t = 0$.

a. Find the acceleration of the particle at time $t = 3$. Is the speed of the particle increasing or decreasing at time $t = 3$? Why or why not?

b. Find all times t in the open interval $0 < t < 7$ at which the particle changes direction?

c. Find the position of the particle at time $t = 3$.

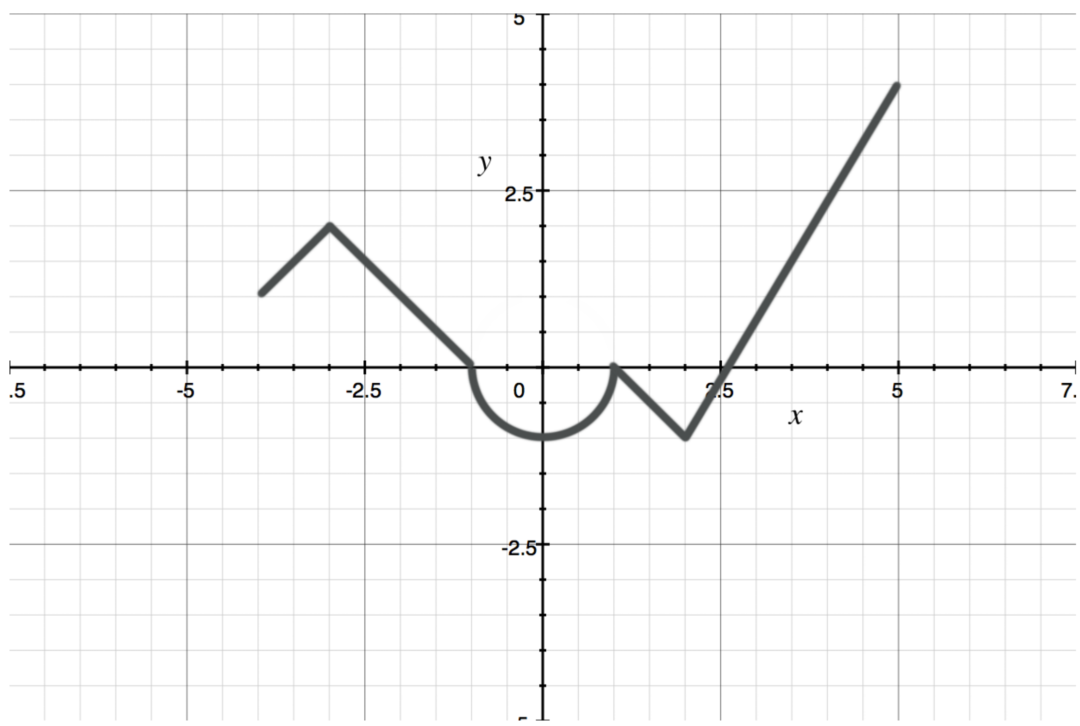
END OF PART A, SECTION II

SECTION II, PART B

Time - 60 **Minutes**

Number of problems - 4

NO CALCULATOR IS ALLOWED FOR THESE PROBLEMS.



3. The graph of the function f shown above consists of a semicircle and four line segments. Let g be the function given by $g(x) = \int_{-3}^{x} f(t)\, dt$.

a. Find $g(-2)$, $g'(-2)$, and $g''(-2)$.

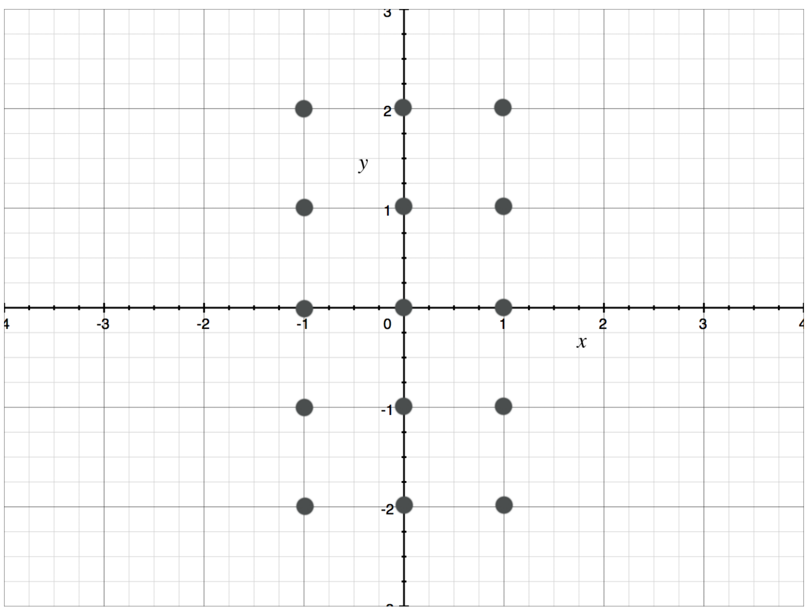
b. Does g have a relative minimum, a relative maximum, or neither at $x = 1$? Explain your answer.

c. Find all values of x in the open interval $(-4,5)$ at which g attains a relative minimum. Explain your answer.

d. Find all values of x in the open interval $(-4,5)$ a which the graph of g has a point of inflection.

4. Consider the differential equation $\frac{dy}{dx} = -\frac{3x^3}{y^3}$.

a. Using the axes given, sketch a slope field for the given differential equation at the twelve points indicated.



b. Let $y = f(x)$ be the particular solution to the differential equation with the initial condition $f(1) = 2$. Write an equation for the line tangent to the graph at $(1,2)$ and use it to approximate $f(1.1)$.

c. Find the particular solution $y = f(x)$ to the given differential equation with the initial condition $f(1) = 2$.

x	1	2	5	7	8	12
f(x)	2	0	-2	3	7	5

5. Let f be a function that is twice differentiable for all real numbers. The table above gives values of f for selected points in the closed interval $1 \leq x \leq 12$.

a. Estimate $f'(6)$. Show your work.

b. Evaluate $\int_1^{12} 2f'(x) - 5\ dx$. Show your work.

c. Use the left Riemann sum with subintervals indicated by the table to approximate $\int_1^{12} f(x)\ dx$.

d. Suppose $f'(5) = 2$ and $f''(x) < 0$ for all x in the closed interval $5 \leq x \leq 7$. Use the line tangent to the graph of f at $x = 5$ to show that $f(6) \leq 0$.

x	f(x)	f'(x)	g(x)	g'(x)
1	2	-2	1	2
2	3	4	3	1
3	8	-1	7	3
4	4	3	10	8

6. The functions f and g are twice differentiable for all real numbers. The table above gives values of the functions and their derivatives at selected values of x. The function $k(x) = g(f(x)) + 5$.

a. Explain why there must be a value c for $2 < c < 4$ such that $k'(c) = 1.5$.

b. Let $h(x) = \int_1^{f(x)} g(t)\ dt$. Find the value of $h'(2)$.

c. If g^{-1} is the inverse function of g, write an equation for the line tangent to the graph of $y = g^{-1}(x)$ at $x = 3$.

d. Evaluate $\int_2^4 f''\left(\frac{x}{2}\right)\ dx$.

STOP

END OF EXAM